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Evolutionary Computation Methods in Optimum Structural Design

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ABSTRACT: An optimum structural design problem is posed as a typical nonlinear programming problem with complex objective function(s) and a vast number of problem constraints. Recently, considerable progress has taken place in the development of a number of evolutionary computation methods for the solution of such problems; namely, evolutionary algorithms (genetic algorithms (GAs), evolution strategies (ESs), evolutionary programming (EP)), simulated annealing (SA), scatter search (SS), classifier systems (CSs) and genetic programming (GP). In this paper optimal (minimum weight) structural design problems, with respect to size, shape and topology variables, are explained and some optimum design applications using GA, ES and SA are presented and discussed for structural systems which make use of trusses.

Keywords: Structural optimization, evolutionary computation methods, trusses

ÖZET: Bir yapı optimizasyonu problemi, kompleks amaç fonksiyonları ve çok sayıda kısıtlayıcıları ile, lineer olmayan tipik bir programlama problemi olarak ortaya çıkmaktadır. Son yıllarda, lineer olmayan programlama problemlerinin çözümü için çeşitli evrimsel hesap yöntemlerinin geliştirilmesinde önemli mesafeler alınmıştır. Bunlara örnek olarak; evrimsel algoritmalar (genetik algoritmalar (GAs), evrimsel stratejiler (ESs), evrimsel programlama (EP)), tavlama simülasyonu (SA), dağınık arama (SS), sınıflamalı sistemler (CSs) ve genetik programlama (GP), gösterilebilir.   Bu makalede optimum ağırlık için, kesit, şekil ve topoloji optimizasyonu ele alınmakta, ve çeşitli kafes kirişler için GA, ES ve SA'ın kullanıldığı yapı tasarımı örnekleri verilmektedir.

Introduction

Engineering design is a complex and usually an iterative process. The main complexity arises from the diverse nature of constraints which need to be satisfied by the final design. A systematic treatment of the analysis phase of such an iterative process using computers and finite element analysis has contributed to handle the problem in a very efficient manner complementing the experience, intuition and creativity of the designer. The conventional approach of the design process starts from a preliminary design with fixed parameters which is then analyzed and the result obtained is studied as to the satisfaction of constraints. This process is repeated until all constraints are satisfied. In this procedure obtaining the best possible design is primarily dependent on the experience of the designer. On the other hand, optimum design seeks the best solution in a different manner. Here, at the problem formulation phase certain design parameters are defined as design variables and the optimal values of these variables are sought as to satisfy the problem constraints and also optimizing the objective(s) of the design. Depending on the chosen design variables and the objective function an optimum solution is obtained which does not require the interference of the designer. A question which is left here is whether the solution is a global or a local solution. A global solution is obtained under certain conditions only and thus good near optimum solutions are acceptable. An optimum design problem is thus posed as a nonlinear programming problem in n-design variables as:

$$\begin{aligned} &\text{optimize (minimize/maximize)} \quad f(\mathbf{x}) && (\mathbf{x})^T=(x_1,x_2,...,x_n) \\ &\text{subject to} \quad h_j(\mathbf{x})=0 && j=1,...,m \qquad (1) \\ &\qquad g_k(\mathbf{x})\leq 0 && k=1,...,p \\ &\qquad x_i^l\leq x_i\leq x_i^u && i=1,...,n \end{aligned}$$

where $(\mathbf{x})$ is the vector of design variables, $f(\mathbf{x})$ the objective function, h_j and g_k are equality and inequality constraints, respectively and x_i^l and x_i^u show the lower and upper bounds on design variable x_i.

This paper is concerned with the solution of such a formulation in relation to optimum structural design. Specifically, the use of some emerging evolutionary computational methods, namely, GAs, ESs and SA are discussed. Illustrative examples are given for some typical truss structures.

Optimum Structural Design

The objectives of a structural design problem may be manyfold. The following discussion considers minimum weight design of discrete systems composed of one-dimensional elements connected at certain nodes, i.e., trusses, beams, frames and grids. For such systems, three main type of problems can be identified; size, shape and topology optimization. In size optimization, the geometry of the structure is totally fixed and the design variables are related to cross-sectional parameters,e.g., the cross-sectional areas of truss members, or cross-sectional dimensions of beams or frames. In shape (or configuration) optimization the positioning of certain nodes are considered as design variables. Finally, in topology optimization member connectivity, i.e., existence or non-existence of structural members are involved. The optimum design problem can be formulated to deal with these cases seperately or consider any two of them together, or all three together (a simultaneous optimization problem). For practical purposes size optimization is usually a discrete optimization problem. In most cases designers choose member cross sections from available section profiles. Shape optimization is necessarily continuous. The position of the nodes are allowed to vary between certain limits. Member connectivity is also handled as a discrete problem. Thus a simultaneous optimization problem is a challenging one, since both discrete and continuous design variables are involved. Morover, especially in large scale structures the number of design variables are increased creating a very large design space of a multimodal nature.

Another important issue in structural optimization is the handling of constraints. The usual constraints for framed structures are those defined on element stresses, nodal displacements and stability. Large number of constraints of different characteristics increases the complexity of the computational procedure. Thus an optimum structural design problem with different objectives, large number of design design variables and constraints (especially for large scale structural systems) requires efficient optimization algorithms.

Evolutionary Computation Methods

In this section as representative of emerging evolutionary computation techniques genetic algorithms, simulated annealing and evolution strategies are briefly discussed as solution procedures for the nonlinear programming problem.

Genetic Algorithms

Genetic Algorithms (Holland, 1975) are stochastic search methods which are based on Darwin's theory of 'survival of the fittest' and adaptation, i.e., natural evolution. Thus, a genetic algorithm starts with a randomly chosen population which is evolved using genetic operators of selection, recombination, and mutation which mimic the natural process of evolution. During a typical generation relatively good individuals replace relatively bad individuals. Whether an individual is good or bad is determined by defining a fitness function (a function representing the aim of the search) and as generations proceed the general fitness of the population is expected to increase. Selection between individuals is performed according to its fitness and fitter individuals are favoured to reproduce more as compared to to less fit individuals. Recombination aims to mix the good characteristics of the selected individuals and carry them into the individuals of the next generation. Mutation operator is applied to individuals to change their structures in an arbitrary manner which in turn alters the genetic similarity of a population as generations progress. This iterative process continues until a preassigned number of generations are completed or a convergence criterion is satisfied. Genetic algorithms are unconstrained search techniques. In the existince of constraints which is the case for structural optimization the handling of constraints becomes an issue to be taken care of (Michaelewicz, 1995), (Hasançebi and Erbatur, 2000a).

Simulated Annealing

Simulated annealing (Kirkpatrick et al., 1983) is an optimization technique which stems from the annealing process of physical systems using principles of thermodynamics and statistical mechanics. The method shows a basic similarity to local search methods (hillclimbing methods) in the sense that it employs only a single design in the search for the optimum solution. The important distinction between the two approaches shows itself when the current design is perturbed to get the next design. In local search methods if the new design yields a higher value of the objective function it is automatically rejected, thus the search is necessarily only allowed in the neigbourhood of the starting design, i.e., it is dependent on the starting design. However, in simulated annealing a probability of acceptance is introduced; if the new design provides a lower value of the objective function its probability of acceptance will be '1' and it will replace the old design, and if the new design results in a higher value of the objective function it is not automatically rejected but will be accepted with a certain probability of acceptance. This mechanism eliminates being stuck to a local optimum and in this sense simulated annealing is a global optimization technique. The acceptance probability is a function of the objective function of the existing and new designs and a temperature parameter. The optimization process is executed starting from an initial value of the temperature and is repeated by lowering its value up to a small final value controlled by an assumed acceptance probability.

Evolution Strategies

Evolution strategies (Schwefel, 1981), (Back, et al, 1991) are very similar to genetic algorithms in the sense that they also use a population of individuals in the evolution process. The main differences lie in the fact that in evolution strategies recombination operator and mutation preceed the selection operator, and that these are not only applied to design variables but also for certain strategy parameters (standard deviations and rotation angles). Another important characteristics of evolution strategies is that these control parameters do not vary by a deterministic fashion but are self-adaptive. The two mostly used evolution strategies are multimembered and expressed as $(\mu + \lambda)$ – ESs and (μ,λ)-ESs. In the former, the generations start from μ individuals from which λ individuals are reproduced. The selection for μ relatively better individuals for the next generation is performed using this enlarged population consisting of $\mu + \lambda$ individuals. In the latter, from μ individuals λ individuals ($\lambda>\mu$) are reproduced and from these individuals the relatively better μ individuals are selected for the next generation. Thus in (μ,λ)-ESs the life of an individual is restricted with one generation only.

Optimum Design of Truss Structures For Minimum Weight

Consider a planar or a space truss structure composed of e-elements and n-nodes. Let A_i , L_i and r_i, (i=1...,e) represent the cross sectional areas, lengths and unit weights of the elements. The nonlinear programming problem for the general case of simultaneous optimization considering size, shape and topology variables is defined as follows:

minimize $\quad W(A_i , C_j , T_i) = S\ r_i\, L_i A_i\, ,\ i = 1,\ldots,e,\ \ j = 1,\ldots,n$

to determine

$\boldsymbol{A}_{ex1}$, vector for size variables
$\boldsymbol{C}_{nx1}$, vector for shape variables
$\boldsymbol{T}_{ex1}$, vector for topology variables (2)

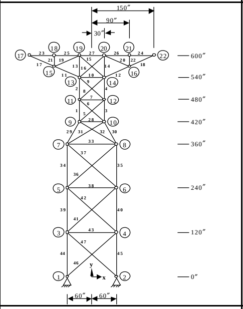
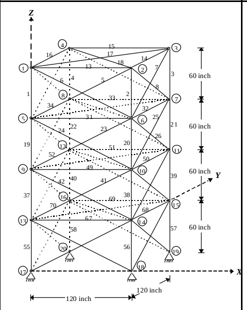
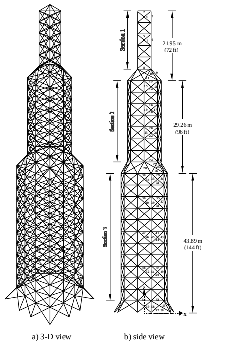
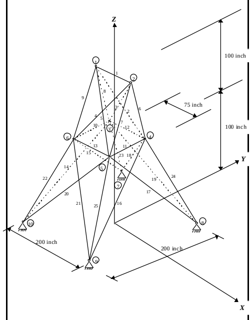
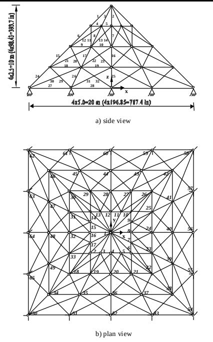
subject to constraints

$\sigma<\sigma_{\text{all}}$ element stresses
$\lambda<\lambda_{\text{all}}$ element slenderness ratios
$u<u_{\text{all}}$ nodal displacements

where the subscript ‘all’ indicates allowable values.

The above discussed evolutionary computational methods are all designed for unconstrained optimization. On the other hand the minimum weight design of trusses (in general problems in structural optimization) includes a large number of constraints related to structural behaviour and other side constraints, e.g., imposed by architectural

Table 1: Some Optimum Design Applications

Structure	No. of Members	No. of Nodes	METHOD			OPT. TYPE		
			GA	SA	ES	S^1	$S+C^2$	$S+C+T^3$
 150" 90" 30" 600" 540" 480" 420" 360" 240" 120" 0" 60" 60"	47	22	a	a	a	-	a	a
 Z 60 inch 60 inch 60 inch 60 inch Y X 120 inch 120 inch	72	20	a	a	a	a	-	-
 Section 1 21.95 m (72 ft) Section 2 29.26 m (96 ft) Section 3 43.89 m (144 ft) a) 3-D view b) side view	942	244	-	a	-	a	-	-
 Z 100 inch 75 inch 100 inch Y 200 inch 200 inch X	25	10	a	a	a	a	-	-
 4x5.0=20 m (4x196.85=787.4 in) a) side view b) plan view	224	65	a	a	-	-	-	a

S^1: Size; C^2: Configuration (shape); T^3: Topology considerations. There are several methods proposed for handling the constraints (Hasançebi and Erbatur, 2000a), (Michaelewicz, 1995). A popular one is based on the use of penalty functions where individuals (designs) which violate constraints are penalized. The penalty function is integrated to the objective function (*W*) and a new modified objective function (*W_m)* is defined, i.e., the constrained problem is transformed into an unconstrained problem:

$$W_m = W + Penalty \tag{3}$$

For minimization problems, there is no penalty if constraints are not violated, otherwise the penalty is a positive value the magnitude of which depends on the severity of the violation; higher the violation higher is the penalty.

Size optimization (Erbatur et al, 2000) (Rajeev and Krishnamoorty, 1992) is carried out for fixed shape and topology. For practical purposes it is a discrete optimization problem. The designer has to select a section from an available profile list. In the lack of efficient discrete optimization techniques, the usual conventional approach was to obtain a continuous solution, then to round it up to the highest existing value. This of course spoils the main idea of optimization. An important contribution of evalutionary computation techniques lie here, since they can deal with discrete , continuous and mixed problems very efficiently. In dealing with size and/or shape and/or topology variables usually a multilevel optimization procedure is followed (Dobbs and Felton, 1969) (Hajela, et al, 1993). Here, firstly a topology optimization is carried out to find the optimum topology or topologies. Then, the size and/or shape design variables are introduced as a second level of optimization. Complete simultaneous optimization problem solutions are given in (Rajan, 1995), (Hasançebi and Erbatur, 2001) and (Hasançebi and Erbatur, 2002).

Some Applications and Discussion

Trusses find applications in a variety of structural systems including but not limited to buildings, bridges, roofs, towers, cranes, antennas, and transmission towers. Existing literature covers many examples including some benchmark truss structures which are used to test the success or efficiency of newly proposed techniques and also, optimum design of large-scale structures composed of complex geometries and a very large number of elements. Some examples of truss design problems which are optimized using GA, SA and ES are given in Table 1. It has been shown that with the use of evolutionary computation techniques, a) better optimum designs are obtained, b) many optimum design problems which were difficult or impossible to be handled by traditional optimization techniques can now be treated efficiently and with confidence. Ongoing research studies in the Department of Civil Engineering at METU recently resulted in computer programs SSTOGA (Size-Shape-Topology Optimization Using GAs), (Hasançebi and Erbatur, 2001) and SSTOSA (Size-Shape-Topology Optimization Using Simulated Annealing), (Hasançebi and Erbatur, 2000b) which are capable to treat size and/or shape and/or topology optimization of 2-D and 3-D trusses considering also important practical situations. Endavours to increase the computational efficiency and wider recognition of evolutionary computation techniques in structural design applications is continuing and it is believed that the outcome will be of significant use to structural designers seeking better designs.

Conclusion

In truss optimization three categories of problems can be identified; (i) sizing, (ii) shape or configuration and (iii) topology optimization. These problems can be handled separately, however the most efficient optimization problem considers these simultaneously. Furthermore, for practical purposes important issues are (a) for size optimization discrete solutions should be favoured, (b) certain important nodes and elements should exist in the final optimized structure, (c) stress, deflection and stability considerations should be included as constraints. Recent studies have revealed that the relatively new evolutionary computation techniques can handle these problems in a much more systematic manner as compared to conventional approaches and thus are proved to be indispensible modern techniques to serve the structural optimization community.

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