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Joint Mechanics and Factors Affecting the Joint Shear Strength

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ABSTRACT: In this study a new design equation for predicting the shear strength of monotonically loaded exterior beam column joints is proposed. The design equation suggested has three differences from the previously proposed equations. First, the equation proposed considers the influence of beam longitudinal reinforcement ratio, which was not taken into account in previously suggested design equations. Second, as the influence of this parameter is taken into account, a more realistic estimate of the influence of joint aspect ratio is obtained. Third, the influence of stirrups is considered differently for joints with low, medium and high amount of stirrup ratios, in a way, which was not considered in previously suggested equations. The results showed that the proposed design equation predicts the joint shear strength of exterior beam column connections accurately with minimal standard deviation and is more reliable than the previously suggested equations. The principles of mechanics is applied on joints and stress and strain analysis is carried out. It is apparent that the proposed equation is in agreement with the predictions of joint mechanics.

Keywords : Beam-column joints, joint shear strength, stress analysis

ÖZET: Bu çalışmada çok sayıda kolon-kiriş birleşimi deneyinden bir veri-tabanı oluşturulmuş, bu veri-tabanı üzerinde parametrik çalışmalar yapılmış ve kolon-kiriş birleşimlerinin kesme dayanımlarını belirleyen yeni bir tasarım denklemi geliştirilmiştir. Bu denklem veri-tabanı üzerinde uygulanmış ve birleşim bölgelerinin kesme dayanımını mühendislik açısından yeteri yakınsaklıkla belirlediği görülmüştür. Daha sonra mekaniğin yerleşmiş prensipleri kolon-kiriş birleşimlerine uygulanmış ve buradan elde edilen veriler ile tasarım denklemi karşılaştırılmıştır. Önerilen tasarım denklemi, mekaniğin temel prensiplerinin birleşim bölgelerine uygulanmasıyla elde edilen bulgular ile uyumludur.

Introduction

It is now generally believed that beam-column joints can be critical regions in reinforced concrete frames under severe seismic effects. Beam-column joint failures have been commonly observed in recent earthquakes worldwide (Bakır and Boduroğlu, 2002a). During the past forty years, significant amount of research has been carried out on seismic behavior of beam-column joints all over the world. However, compared to cyclically loaded joints, little information exists in literature for predicting the shear strength of monotonically loaded exterior joints. In a previous paper, the authors proposed a new design equation for predicting the shear strength of monotonically loaded exterior beam-column joints (Bakır and Boduroğlu, 2002b, Bakır and Boduroğlu, 2002c). The aim of this paper is to compare the new design equation with the principles of joint mechanics.

The authors carried out a parametric investigation of exterior beam-column joint behavior based on 58 tests conducted in the Europe. Table 1 shows the experimental database used in this study. The database comprises of the tests of Ortiz (1993), Kordina (1984), Scott (1992), Scott & Hamill (1998), Taylor (1974), and Parker & Bullman (1997). A typical specimen in the experimental database is given in Figure 1.

Development of the design equation

In a previous study (Bakır and Boduroğlu, 2002b), several parametric studies are carried out on the experimental database in Table 1. The results showed that the joint shear strength is independent of the column longitudinal reinforcement ratio and column axial stress and is influenced by factors such as beam longitudinal reinforcement ratio, stirrups and the joint aspect ratio. As a result of these parametric studies, Eq.(1) is proposed for the design of monotonically loaded exterior beam-column joints.

$$V_j = \frac{\left(\frac{h_c}{h_b}\right)^{0.61}}{0.71\ \beta\ \gamma\left(100 * \frac{A_{sb}}{b_b d_b}\right)^{0.4289}\left(\frac{2}{b_c + b_b}\right) h_c \sqrt{f_c}} + \alpha\ A_{sje} f_y \qquad (1)$$

where α = 0.664 for joints with low amount of stirrups; α = 0.6 for joints with medium amount of stirrups and α = 0.37 for joints with high amount of stirrups.

A_{sje} is the area of the stirrups

f_y is the stirrup yield strength

β = 0.85 for joints detailed by U bars and β = 1 for joints detailed by L bars.

γ = 1.37 for inclined bars in the joint and γ = 1 for others.

A_{sb} = Total area of beam reinforcement

b_b = the breadth of the beam

d = the depth of the beam

In this study, the physical interpretation of Eq.(1) proposed by the authors will be explained using the established principles of joint mechanics.

Comparison of the proposed design equation with the established principles of joint mechanics

In order to investigate the reliability of the design equation, the authors investigated the established equations on the basic mechanics of reinforced concrete beam-column joints. This has been also previously discussed by Paulay (1986) and by Bonacci & Pantazopoulou for interior joints (1992) who have also taken into account the joint deformations. The typical loading system considered in analysis of exterior beam-column joints is shown in Fig. 2. Both of the authors use the average stresses for equilibrium as shown in Fig.3. Fig. 3 depicts the equilibrium of vertical and horizontal forces. Figure 2 shows that equilibrium of forces in the horizontal direction require the average transverse compressive stress in the joint σ_x defined as:

$$\sigma_x = -\frac{A_{sb}}{b_{eff} h_b} f_s - \frac{A_{sje}}{b_{eff} h_b} f_w \qquad (2)$$

where f_s is the average stress in the beam reinforcement,
f_w is the average stress in the transverse reinforcement
Consequently, the average normal concrete stress in the *y* direction σ_y can be expressed as :

$$\sigma_y = -\frac{A_{scol}}{b_{eff} h_c} f_{scol} - \frac{N}{b_{eff} h_c} \qquad (3)$$

where f_{scol} is the average stress in the column reinforcement
N is the column axial load
Defining the average joint shear stress in the joint as τ_{av}, the maximum principal stress associated with the stress tensor is given as;

$$\sigma = \begin{bmatrix} \sigma_x & \tau_{av} & 0 \\ \tau_{av} & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix} \qquad (4)$$

where σ_z is the confining stress provided by stirrups in the *z* direction.

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0 \qquad (5)$$

In order to determine the principal stresses, Eq.5 has to be solved;
where $I_1=\sigma_x+\sigma_y+\sigma_z$
$I_2=\sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_x\sigma_z-\tau_{av}^2$
$I_3=\sigma_x\sigma_y\sigma_z-\sigma_z\tau_{av}^2$

The tensile stress in the concrete is negligible and therefore $\sigma_1=0$, which consequently gives ;

$$\sigma_y = \frac{\tau_{av}^2}{\sigma_x} \qquad (6)$$

From the Mohr's circle,

$$\tan 2\theta = \frac{2\tau_{av}}{\sigma_x - \sigma_y} \qquad (7)$$

If Eq.(6) is substituted into Eq.(7), the following quadratic equation ensues;

$$\tau_{av}^{2} + \left(\tan\theta + \frac{1}{\tan\theta} \right) \sigma_x \tau_{av} - \sigma_x^{2} = 0 \tag{8}$$

which gives;

$$\tau_{av} = -\frac{\sigma_x}{\tan\theta} \tag{9}$$

Using equation 6, we have;

$$\sigma_y = -\frac{\tau_{av}}{\tan\theta} \tag{10}$$

Collins and Mitchell (1991) suggest the following equation for the maximum stress in concrete panels;

$$f_{2\max} = \frac{f_c}{0.8 + 170\varepsilon_1} \langle f_c \tag{11}$$

The principal compressive stress is given by;

$$\sigma_2 = \left(2\left(\frac{\varepsilon_2}{-0.002}\right) - \left(\frac{\varepsilon_2}{-0.002}\right)^2 \right) f_{2\max} \tag{12}$$

σ_2 is also given from Mohr’s circle as;

$$\sigma_2 = \sigma_x + \sigma_y = -\tau_{av}\left(\tan\theta + \frac{1}{\tan\theta}\right) \tag{13}$$

Thus the average joint shear stress can be expresses as;

$$\tau_{av} = -\frac{\sigma_2}{\left(\tan\theta + \frac{1}{\tan\theta}\right)} \tag{14}$$

Equations 11 to 14 show very clearly that as the principal tensile strain increases, the average joint shear stress decreases. Thus it is necessary to express the principal tensile strain in terms of the strains in the *x* and *y* directions in order to investigate the factors that influence the joint shear strength. From Mohr’s circle, it is known that;

$$\tan 2\theta = \frac{\gamma}{\varepsilon_x - \varepsilon_y} \tag{15}$$

From Mohr’s circle, the principal tensile strain will be;

$$\varepsilon_1 = \frac{(\varepsilon_x + \varepsilon_y)}{2} + \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2} \tag{16}$$

If Equation 15 is substituted into Eq.(16) and appropriate trigonometric transformations are carried out, Eq.(17) given by Bonacci and Pantazopoulou is obtained.

$$e_1 = (\frac{e_x - e_y \tan^2 q}{1 - \tan^2 q}) \tag{17}$$

The next step will be to express the strains in the *x* and *y* directions in terms of the stresses.

$$s_x = -t_{av} \tan q = -\frac{A_{sb} f_s}{b_{eff} h_b} - \frac{A_{sje} f_w}{b_{eff} h_b} = -(\frac{A_{sb}}{b_{eff} h_b} m + \frac{A_{sje}}{b_{eff} h_b}) f_w \tag{18}$$

where $m = f_s / f_w$

The strain in the *x* direction can therefore be expressed as;

$$e_x = \frac{f_w}{E_s} = \frac{t_{av} \tan q}{E_s (\frac{A_{sb} m}{b_{eff} h_b} + \frac{A_{sje}}{b_{eff} h_b})} \tag{19}$$

The strain in the *y* direction can similarly be expressed as;

$$e_y = \frac{f_{scol}}{E_s} = (\frac{t_{av}}{\tan q} - \frac{N}{b_{eff} h_c}) \frac{b_{eff} h_c}{A_{scol} E_s} \tag{20}$$

If equations 19 and 20 are substituted into equation 17;

$$e_1 = \frac{1}{E_s (1 - \tan^2 q)} (t_{av} \tan q (\frac{1}{\frac{A_{sb}}{b_{eff} h_b} m + \frac{A_{sje}}{b_{eff} h_b}} - \frac{1}{\frac{A_{scol}}{b_{eff} h_c}}) + \frac{N \tan^2 q}{b_{eff} h_c \frac{A_{scol}}{b_{eff} h_c}}) \tag{21}$$

It is evident from the inspection of experiments that cracks extend throughout the diagonal of the joint. So the angle of principal stresses can be expressed as;

$$\tan q = \frac{h_b}{h_c} \tag{22}$$

If equation 22 is substituted into equation 21,

$$e_1 = \frac{1}{E_s (1 - (\frac{h_b}{h_c})^2)} (t_{av} \frac{h_b}{h_c} (\frac{1}{\frac{A_{sb} m}{b_{eff} h_b} + \frac{A_{sje}}{b_{eff} h_b}} - \frac{1}{\frac{A_{scol}}{b_{eff} h_c}}) + \frac{N (\frac{h_b}{h_c})^2}{A_{scol}} \tag{23}$$

The above equation shows that the principal tensile strain is increased by the joint aspect ratio and column longitudinal reinforcement ratio and the axial load on the column whereas it is decreased by increasing beam longitudinal reinforcement ratio and the stirrup ratio. The shear stress in the joint is dependent on the principal tensile strain as evident from Eqs. (11) and (12). It is therefore evident from Eqs. (17, 18 and 23) that the joint shear strength increases as the beam longitudinal reinforcement ratio and the transverse reinforcement ratio increases. Equation 3 shows that the joint shear strength increases as the column load and the column longitudinal reinforcement increases but Equation 23 shows that as the longitudinal column reinforcement and the column load increases, the principal tensile stresses increase which consequently decreases the normalized joint shear strength. Therefore the increase in the joint shear strength due to Equation 3 is offset by the increase in the principal tensile strain. The above conclusions are totally in accordance with the predictions of the authors' equation. The investigation of joint mechanics confirms the design equation proposed by authors in Eq. (1).

Conclusions

The purpose of this investigation was to study the effect of the parameters influencing the behaviour of beam to column connections and to determine if the proposed design equation is in accordance with the established principles of joint mechanics. From the analysis of the tests and results of the parametric studies, the following design recommendations can be made.

1. The predictions of the proposed equation is in accordance with the joint mechanics.
2. Column axial load has no influence on ultimate shear capacity of the joint.
3. Stirrups increase the joint shear strength.
4. Increasing the beam longitudinal reinforcement ratio increases the joint shear strength.
5. The authors applied their equation on the experimental database in Table 1. The results showed that the average $V_{jpredicted}$ /V_{jtest} values for the authors' equation applied on all the experiments in the experimental database is 0.88 and the standard deviation is 0.1. The results show that the equation suggested gives realistic and conservative estimates of the joint shear strength.

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Figure 1: The typical specimen

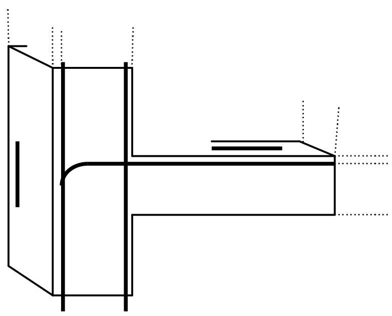

Figure 2: Joint geometry

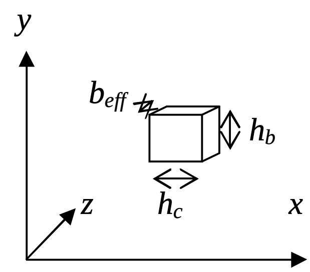

Figure 3: Stress equilibrium

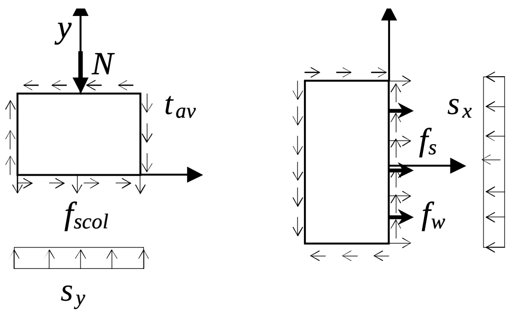

Table 1: The experimental database

Investigator	specimen	detail	hb/hc	beam rein.ratio	fc(Mpa)	column axial load	SI (Mpa)^0.5	Vjpredicted/Vjactual	Failure modes
Ortiz	BCJ 1	L bar	1.33	1.1	34	0	0	0.68	js
	BCJ 2	L bar	1.33	1.1	38	0	0.16	0.77	js
	BCJ 3	L bar	1.33	1.1	33	0	0	0.64	js
	BCJ 4	L bar	1.33	1.1	34	0	0.33	0.78	js
	BCJ 5	L bar	1.33	1.1	38	300	0	0.72	js
	BCJ 6	L bar	1.33	1.1	35	300	0	0.68	js
	BCJ 7	L bar	1.33	1.1	35	300	0.76	0.68	b
Kordina	RE 2	L bar	2.00	0.9	25	240	0	0.66	js
	RE 3	L bar	1.50	1.8	40	400	0.26	0.96	js
	RE 4	L bar	1.50	1.2	32	51	0.19	0.83	js
	RE6	L bar	1.50	1.2	32	213	0.38	0.91	js
	RE7	L bar	1.40	1.3	26	650	0.43	0.87	js
	RE8	U bar	1.40	1.3	28	525	0.42	0.90	js
	RE9	U bar	1.40	1.3	28	770	0.41	0.86	js
	RE10	U bar	1.56	1.2	24	551	0.45	0.94	js
Taylor	P1/41/24	L bar	1.43	2.4	33	240	0.3	0.97	js
	P2/41/24	L bar	1.43	2.4	29	240	0.3	0.94	js
	P2/41/24A	L bar	1.43	2.4	47	240	0.26	0.92	js
	A3/41/24	L bar	1.43	2.4	27	240	0.3	0.88	js
	D3/41/24	L bar	1.43	2.4	53	60	0.24	0.89	js
	B3/41/24	L bar	1.43	2.4	22	240	0.75	0.92	js
	C3/41/24BY	U bar	1.43	2.4	32	240	0.31	1.04	js
	C3/41/13Y	U bar	1.43	1.4	28	240	0.33	0.95	js
Scott	C1AL	L bar	1.40	1.1	33	50	0.188	0.87	js
	C4	L bar	1.40	2.1	41	275	0.203	0.89	js
	C4A	L bar	1.40	2.1	44	275	0.196	0.86	js
	C4AL	L bar	1.40	2.1	36	50	0.218	0.86	js
	C7	L bar	2.00	1.4	35	275	0.22	0.90	js
	C3L	U bar	1.40	2.1	35	50	0.22	1.03	js
	C6	U bar	1.40	2.1	40	275	0.21	1.05	js
	C6L	U bar	1.40	2.1	46	50	0.19	0.94	js
	C9	U bar	2.00	1.4	36	275	0.22	0.93	js
Scott & Hamil	C4ALNO	L bar	1.40	2.1	42	50	0	0.88	p
	C4ALN1	L bar	1.40	2.1	46	50	0.229	0.85	js
	C4ALN3	L bar	1.40	2.1	42	50	0.478	0.78	js
	C4ALN5	L bar	1.40	2.1	50	50	0.718	0.85	js
	C4ALHO	L bar	1.40	2.1	104	100	0	0.86	p
	C6LNO	U bar	1.40	2.1	51	50	0	0.92	js
	C6LN1	U bar	1.40	2.1	51	100	0.19	0.96	js
	C4ALH1	L bar	1.40	2.1	95.2	100	0.159	0.93	b
	C4ALH3	L bar	1.40	2.1	105.6	100	0.302	0.97	b
	C4ALH5	L bar	1.40	2.1	98.4	100	0.469	1.00	b
	C6LN3	U bar	1.40	2.1	49	50	0.44	0.92	js
	C6LN5	U bar	1.40	2.1	37	50	0.765	0.74	js
	C6LHO	U bar	1.40	2.1	101	100	0	0.72	js
	C6LH1	U bar	1.40	2.1	102	100	0.153	0.98	js
	C6LH3	U bar	1.40	2.1	97	100	0.472	0.93	js
parker	4a	L bar	1.67	0.9	39	0	0	-	c
	4b	L bar	1.67	0.9	39	300	0	1.05	js
	4c	L bar	1.67	0.9	37	600	0	0.83	js
	4d	L bar	1.67	0.9	39	0	0	0.97	js
	4e	L bar	1.67	0.9	40	300	0	0.92	js
	4f	L bar	1.67	0.9	38	600	0	0.78	js
	5a	L bar	1.67	0.9	42	0	0.404	-	c
	5b	L bar	1.67	0.9	43	300	0.4	1.08	js
	5d	L bar	1.67	1.4	43	0	0.6	-	c
	5e	L bar	1.67	1.4	45	300	0.589	-	c
	5f	L bar	1.67	1.4	43	600	0.6	0.86	js
							average	0.88	
							standard deviation	0.10	