ECAS2002 International Symposium on Structural and Earthquake Engineering, October 14, 2002, Middle East Technical University, Ankara, Turkey

Time Domain Response of Multidegree-of-Freedom Systems with Fuzzy Characteristics to Seismic Action

A.A. Kasimzade
Ondokuz Mayıs University, Faculty of Engineering, Department of Civil Engineering, 55139 Samsun/Turkey; azer@omu.edu.tr

ABSTRACT: Structure with finite dynamic degree of freedom and fuzzy geometrical and physical parameters are analyzed under earthquake action, membership functions of quality characteristics have been established, deflections of the fuzzy results as rates of the deterministic are estimated with percents, safety criterion and its estimating is given. Experimental test is made by using orthogonal array Taguchi's method. For each α – cut level the equations of motion is solved using step by step integration in the time domain by the Numerical Wilson-θ's method.Some operations with fuzzy parameters are provided by using extension principle (Zadeh,1965). Numerical examples are given and the results are explained.

Keywords : Earthquake action; fuzzy set; Taguchi method; finite element analysis

ÖZET: Bulanık geometrik ve fizik parametrelere sahip sonlu dinamik serbestlik dereceli (d.s.d.) yapının deprem etkisine analizi yapılmıştır. Karakter niceliklerin üyelik fonksiyonları kurulmuş, deterministik sonuçlara oranla sapmaları yüzdelerle değerlendirilmiştir,emniyet kriteri ve değerlendirilmesi verilmiştir. Deneme planlaması ortogonal alan (Taguchi) metodu ile yapılmıştır. Her bir α-kesim için hareket denklemleri zaman-tanım alanında sayısal (Wilson-θ) metodu ile çözülmüştür. Bulanık parametreler üzerinde bir sıra işlemler genişleme prensibine (Zadeh,1965) dayanarak yapılmıştır. Sayısal örnekler verilmiştir, sonuçlar açıklanmıştır.

Introduction

As is known, the analysis under seismic action of special structures necessitates the use of actual records or simulated acceleration of ground. In this analysis, one of crucial points is, the estimation of sensitivity of reaction of a structure, with respect to the uncertainties in geometrical and physical parameters (height of a floor, mass, rigidity, damping etc....). In the applications uncertainties are classified as being random and fuzzy. In the submitted work in a direction of each dynamic degree of freedom (d.d.f) mass, height of a floor, module of elasticity is accepted by fuzzy quantities, with treangular membership function and are replaced by the α - cuts. Thus, for every α -cut is constructed the equation of motion. Despite of the availability of investigations on fuzzy equations (Buckley,1992), practical methods for the applications is rather limited. In some stages numerical realization of a considered (examined) problem, use of interval arithmetics for every α -cut (based on a principle of expansion Zadeh) is very difficult to be realized, because borders of reactions (target parameters) of the system are undefined. In this study the marked difficulties of transformation connected to the interval analysis have been overcome for every α -cut by application the Taguchi method .Thus factors on whose variability the system is sensitive to, is replaced for each α -cut and for each stage of a experiment combination of constructed equation that is solved by a numerical Wilson–θ method. Membership functions for the system (structure) reactions, deviation with respected to the deterministic values, combination number equvalent to system critical state and equvalent time diring the effective earthquake interval have been obtained.

Related to this study , random vibration of the system with fuzzy parameters in one d.d.f system application has been investigated in (Wang,1992) ; eigen vector and eigen value problems for the multi degree of freedom systems with fuzzy parameters has been investigated in (L.Chen,1997) and (B.Lullemend,1999) respectively. Investigation of system reactions coupling fuzzy logic with statistics has been investigated in (Wadia – Foscetti,2000). Estimation of the relibilaty of the structure foundation – ground (with stochastic characteristics) systems under seismic action has been investigated in (Kasumov,1999).

Formulation

More sensitive fuzzy parameters of reactions of structure, namely height of floor (L_i), elasticity module of columns (E_i), mass (m_i) in the direction of i-th d.d.f. are accepted to have two level triangular membership functions μ(x) .

$$m(x) = \begin{cases} 0, & if\ x \le x_L\ or\ if\ \ x_R \le x \\ f_1(x), & if\ x \ge x_L \quad x \le x_P \\ f_2(x), & if\ \ x \ge x_P \quad x \le x_R \end{cases} \qquad (1)$$

$$f_1(x) = \frac{x - x_L}{x_P - x_L}$$

$$f_2(x) = \frac{x_R - x}{x_R - x_P}$$

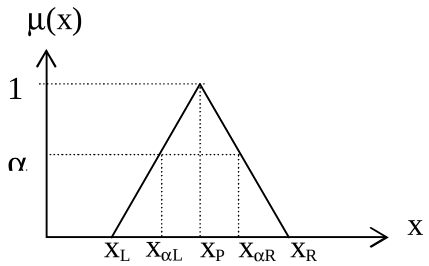

Figure 1. Fuzzy number with triangular membership function and its α-cut

After that, parameters are fuzzyfied, using full factorial and orthogonal arrey testing plan for each α-cut level (the rigidity of story $K_i(E_i,L_i)$ is periphery related with E_i and L_i factors; only for the imbricated storeys (d.d.f.), interaction of the rigidity factors are accepted).

A fuzzy relation f (AxB) from fuzzy set $X = \left[\frac{m_X(x)}{x}\right]$ to a fuzzy set $Y = \left[\frac{m_Y(y)}{y}\right]$ is a fuzzy subset of the Cartesian product XxY, which is a mapping from X to Y. A fuzzy relation f (AxB) is expressed as (Zadeh's extension principle):

$$f(AxB) = \sum_{x \in X} \sum_{y \in Y} \frac{\min(m_A(x), m_B(y))}{f(x, y)} \quad (2)$$

Fuzzy arithmetic is based on the Zadeh's extension principle (2). The computational features of the extension principle can be achieved by using the α-cut representation of fuzzy numbers.

For each α-cut level fuzzy parameters for the testing combination number according to the critical situation, in interaction between two storeys (d.d.f.) equation of motion are errange as:

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [K]\{x\} = \{F_*\} \quad ;$$
$$F_{i*} = -m_i \ddot{x}_g(t) \quad (3)$$

Where [m],[c],[K] are mass, rigidity and damping matrix of system, if the problem is aimed to be solved using the finite element method, they are expressed in the form :

$$[m] = \sum [m_i]$$
$$[K] = \sum [K_i] \quad (4)$$

Where $[m_i]$, $[K_i]$ are mass and rigidity of the systems in the i-th d.d.f. When given vector {ξ} with damping ratio elements for the each d.d.f., damping coefficient matrix is expressed as:

$$[c] = [m] \sum_{i=1,n} a_i \left([m]^{-1}[K]\right)^i \quad (5)$$

$$\{a\} = [w]^{-1} 2\{x\};$$

$$\{x\} = (x_1 x_2 ....x_n)^T$$

$$[w] = \begin{bmatrix} w_1 & w_1^3 & w_1^5 & . & . & w_1^{2n-1} \\ w_2 & w_2^3 & w_2^5 & . & . & w_2^{2n-1} \\ . & . & . & . & . & . \\ w_n & w_n^3 & w_n^5 & . & . & w_n^{2n-1} \end{bmatrix}$$

Where ω_i , frequency of structure for i-th vibration mod ; n , total d.d.f. of system.
$\{F_*\}$, F_{*i} =$- m_i \ddot{x}_g(t)$, $\ddot{x}_g(t)$, dynamic force in the i-th d.d.f. of the system evaluated under ground acceleration.

Solution

For each α-cut of the fuzzy parameters testing combination number accepted to the critical position of the storey interaction, the equation of motion (3) is solved using step by step integration in the time domain using the Wilson-θ's method.As a critical situation ,the time,for the maximal overturning moment in the base of structure (in n_e-th testing $t_{*e} = t_{max\ (Mov)}$) is accepted. In critical situation $e_{No,*}$ (t_{*e}), for each α-cut of fuzzy parameters,response of the system is expressed in the form:

$$u_e, \dot{u}_e, \ddot{u}_e, F_{ue}(:, n_e) = u, \dot{u}, \ddot{u}, F_u(:, e_{No,*}) \tag{6}$$

Where the designation u(:,j) means j-th column of a matrix u.

For the n_e=1-n_{ex} number testing combination, displacement (*u*), velocity $(\dot{u})$, acceleration $(\ddot{u})$, seismic force (F_u), overturning moment (M_{ov}),..are expressed in the form:

$$u_e, \dot{u}_e, \ddot{u}_e, F_{ue} \ldots\ldots = \left.\underbrace{\begin{bmatrix} \quad & & \quad \\ & & \\ & & \end{bmatrix}}_{n_{ex} = number\ of\ testing\ combination}\right\} k = number\ of\ d.d.f. \tag{7}$$

$$M_{OV} = \overbrace{[\qquad\qquad\qquad\qquad]}\ \}1row \tag{8}$$

Right ($M_{ov,R}$) and left ($M_{ov,L}$) values of the overturning moment and numbers of testing for this values $e_{No,L}$, $e_{No,R}$ are found for i_i-th α-cut, based on extension principle:

$$\begin{aligned} [M_{ov,L}, e_{No,L}] &= \min(M_{ov,e}) \\ [M_{ov,R}, e_{No,R}] &= \max(M_{ov,e}) \end{aligned} \tag{9}$$

Response of the structures (displacement, velocity, acceleration,...) which correspond to testing numbers $e_{No,L}$, $e_{No,R}$ for the left and right values for each i_i-th α-cut level are evaluated as (the designation u(:,[i j]) means i-th and j-th columnes of a matrix u) :

$$u_{i_i,e}(:,[2(i_i-1)\quad 2i_i]) = [\underbrace{u_e(:,e_{No,L})}_{left}\quad \underbrace{u_e(:,e_{No,R})}_{right}] \tag{10}$$

....

$$M_{ov,i_i,e}(:,[2(i_i-1)\quad 2i_i]) = [\overbrace{M_{ov,L}}^{left}\quad \overbrace{M_{ov,R}}^{right}] \tag{11}$$

After i_i=1- m_{ns} time realization matrices (10), (11) is expressed in the form:

$$\left.\begin{matrix} u_{i_i,e} \\ \dot{u}_{i_i,e} \\ \ddot{u}_{i_i,e} \\ F_{u,i_i,e} \\ \cdots \end{matrix}\right\} = \left.\underbrace{\begin{bmatrix} \overbrace{\quad}^{1st.\alpha-section} & \overbrace{\quad}^{2.nd\ \alpha-section} & & \overbrace{\quad}^{m_{ns}.\alpha-section} \\ \cdot\ \cdot & \cdot\ \cdot & & \cdot\ \cdot \\ \cdot\ \cdot & \cdot\ \cdot & & \cdot\ \cdot \\ \cdot\ \cdot & \cdot\ \cdot & & \cdot\ \cdot \\ \cdot\ \cdot & \cdot\ \cdot & & \cdot\ \cdot \\ \cdot\ \cdot & \cdot\ \cdot & & \cdot\ \cdot \end{bmatrix}}_{m_{ns}\,x\,2\quad number\ columns}\right\} k\quad number\ d.d.f. \tag{12}$$

$$M_{ov,i_i,e} = \overbrace{[\qquad\qquad\qquad\qquad\qquad\qquad]}\ \}1row \tag{13}$$

According to these results, the membership functions of the system response and its deflections regarding deterministic response, are calculated easily.

Numerical Analyses

The shear building shown in the fig.2 with its physical and geometrical parametres given below is investigated:

$L_1 = (L_{1L} = 0.09L_{1P} \quad L_{1P} = 4.572 \quad L_{1R} = 1.01L_{1P});(metre)$
$L_2, L_3, L_4, L_5 = (L_{2L} = 0.09L_{2P} \; L_{2P} = 3.0481 \; L_{2R} = 1.01L_{2P});$
$m_1 = (m_{1L} = 0.95m_{1P} \; m_{1P} = 23826.8 \; m_{1R} = 1.05m_{1P});(Newton)$
$m_2, m_3, m_4, m_5 = (m_{2L} = 0.95m_{2P} \quad m_{2P} = 11563.0 \quad m_{2R} = 1.05m_{2P});$
$J_1, J_2, J_3, J_4, J_5 = (2J_0 \, 2J_0 \, 2J_0 \;\; 2J_0 \;\; 2J_0)^T;$
$J_0 = 2.0695.10^{-4} m^4$
$\{x\} = (0.05 \quad 0.05 \quad 0.05 \quad 0.05 \quad 0.05)^T$
$E_i = [E_{iL} = 0.95E_{iP} \qquad E_{iP} = 1.0349 \;\; 10^{11} \qquad E_{iR} = 1.05E_{iP}];$
$(N/m^2);(i = 1-5)$

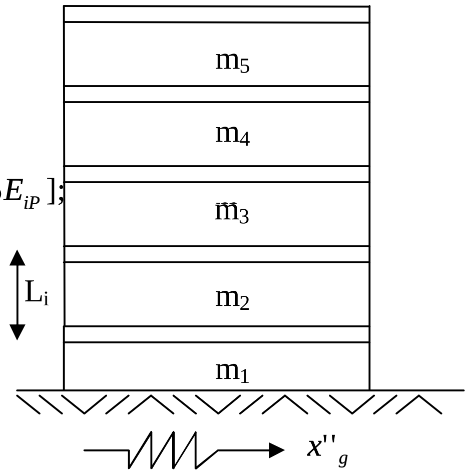

Figure 2. 5 story shear building

First 10.19 seconds records for the East-West component of 1940 El Centro earthquake is accepted as ground acceleration. Integration time step:Δt=0.02, parameter of θ=1.4 (method becomes unconditionally stable) is accepted.

Maximum changing interval characteristic reactions of the system with fuzzy parameters L(standart deviation from its average=1%), E(standard deviation from its average=5%),m(standard deviation from its average=5%)are given in Table4(case "A").

Maximum changing interval characteristic reactions of the system with fuzzy parameter L(standard deviation from its average=1% ; case "B"), the only elasticity module (standard deviation from its average=5%; case "C"), the only story masses (standard deviation from its average=5%; case "D") is given in Tables 5, 6, 7 respectivly.

The sensitivity of the structure to changes in the fuzzy parameters L, E, m for the overturning moment (comparing A,B,C,D cases) are shown below:

		Deviation of Fuzzy overturning moment regarding to deterministic results (for the left and right deviation values in percent)		Testing Combination No for the $M_{ov,L}$ and $M_{ov,R}$ state respectively
Case A(L(1%),E(5%),m(5%))	;	%7.03 ≤ M_{ov} ≤%13.14	;	e_L =1 e_R = 5
Case B(L(1%))	;	%1.9314≤ M_{ov} ≤%7.08	;	e_L =1 e_R = 3
Case C(E(5%))	;	% 0.99≤ M_{ov} ≤%13.27	;	e_L =1 e_R = 2
Case D(m(5%))	;	%5.73≤ M_{ov} ≤%8.24	;	e_L =1 e_R = 2

That for this type structure, is a remarcable feature is more sensitive to change in the story height (case "B").